DGP Cosmology with a Non-Minimally Coupled Scalar Field on the Brane

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Abstract

We construct a DGP inspired braneworld scenario where a scalar field non-minimally coupled to the induced Ricci curvature is present on the brane. First we investigate the status of gravitational potential with non-minimal coupling and observational constraints on this non-minimal model. Then we further deepen the idea of embedding of FRW cosmology in this non-minimal setup. Cosmological implications of this scenario are examined with details and the quintessence and late-time expansion of the universe within this framework are examined. Some observational constraints imposed on this non-minimal scenario are studied and relation of this model with dark radiation formalism is determined with details.

PACS: 04.50.+h, 98.80.-k
Key Words: Braneworld Cosmology, DGP Scenario, Quintessence Model, Late-time Expansion

1 Introduction

Based on light-curves analysis of several hundreds type Ia supernovae [1,2], observations of the cosmic microwave background radiation by the WMAP satellite [3] and other CMB-based experiments [4,5], it has been revealed that our universe is currently in a period of accelerated expansion. Some authors have attributed this late-time expansion of the universe to an energy component referred to as *dark energy*. The simplest example in this regard is the cosmological constant itself which provides a model of dark energy. However, it is unfavorable since it requires a huge amount of fine-tuning [6]. Phantom fields [7], quintessence [8] and modification of gravitational theory itself [9,10] are other attempts to explain this late time expansion of the universe. In the spirit of the modified gravitational theory, Carroll *et al* have proposed R^{-1} modification of the usual Einstein-Hilbert action [11]. It was then shown that this term could give rise to accelerating solutions of the field equations without dark energy.

On the other hand, theories of extra spatial dimensions, in which the observed universe is realized as a brane embedded in a higher dimensional spacetime, have attracted a lot of attention in the last few years. In this framework, ordinary matters are trapped on the brane but gravitation propagates through the entire spacetime [9,12,13]. The cosmological evolution on the brane is given by an effective Friedmann equation that incorporates the effects of the bulk in a non-trivial manner [14]. From a cosmological view point, the importance of brane models lies, among other things, in the fact that they can provide an alternative scenario to explain the late-time accelerating expansion of the universe.

Theories with extra dimensions usually yield correct Newtonian limit at large distances since the gravitational field is quenched on sub-millimeter transverse scales. This quenching appears either due to finite extension of the transverse dimensions [12,15] or due to sub-millimeter transverse curvature scales induced by negative cosmological constant [13,16-19]. A common feature of these type of models is that they predict deviations from the usual 4-dimensional gravity at short distances. The model proposed by Dvali, Gabadadze and Porrati (DGP) [9] is different in this regard since it predicts deviations from the standard 4-dimensional gravity even over large distances. In this scenario, the transition between four and higher-dimensional gravitational potentials arises due to the presence of both the brane and bulk Einstein terms in the action. In this framework, existence of a higher dimensional embedding space allows for the existence of bulk or brane matter which can certainly influence the cosmological evolution on the brane. Even if there is no 4-dimensional Einstein-Hilbert term in the classical theory, such a term should be induced by loop-corrections from matter fields [20,21]. Generally one can consider the

effect of an induced gravity term as a quantum correction in any brane-world scenario. A particular form of bulk or brane matter is a scalar field. Scalar fields play an important role both in models of the early universe and late-time acceleration. These scalar fields provide a simple dynamical model for matter fields in a brane-world model. In the context of induced gravity corrections, it is then natural to consider a non-minimal coupling of the scalar field to the intrinsic (Ricci) curvature on the brane that is a function of the field. The resulting theory can be thought of as a generalization of Brans-Dicke type scalar-tensor gravity in a brane-world context. In contrast to common belief, the introduction of non-minimal coupling (NMC) is not a matter of taste; NMC is instead forced upon us in many situations of physical and cosmological interest. There are many compelling reasons to include an explicit non-minimal coupling in the action. For instance, NMC arises at the quantum level when quantum corrections to the scalar field theory are considered. Even if for the classical, unperturbed theory this NMC vanishes, it is necessary for the renormalizability of the scalar field theory in curved space. In most theories used to describe inflationary scenarios, it turns out that a non-vanishing value of the coupling constant cannot be avoided. In general relativity, and in all other metric theories of gravity in which the scalar field is not part of the gravitational sector, the coupling constant necessarily assumes the value of $\frac{1}{6}$. The study of the asymptotically free theories in an external gravitational field with a Gauss-Bonnet term shows a scale dependent coupling parameter. Asymptotically free grand unified theories have a non-minimal coupling depending on a renormalization group parameter that converges to the value of $\frac{1}{6}$ or to any other initial conditions depending on the gauge group and on the matter content of the theory. An exact renormalization group study of the $\lambda\phi^4$ theory shows that NMC$= \frac{1}{6}$ is a stable infrared fixed point. Also in the large N limit of the Nambu-Jona-Lasinio model, we have NMC$= \frac{1}{6}$. In the $O(N)$- symmetric model with $V = \lambda\phi^4$, NMC is generally nonzero and depends on the coupling constants of the individual bosonic components. Higgs fields in the standard model have NMC$= 0$ or $\frac{1}{6}$. Only a few investigations produce zero value(for a more complete discussion of these issues we refer to papers by V. Faraoni, specially Ref. [22] and references therein). In view of the above results, it is then natural to incorporate an explicit NMC between scalar field and Ricci scalar in the inflationary paradigm and in quintessence models. In particular it is interesting to see the effect of this NMC in a DGP-inspired braneworld cosmology.

There are several studies focusing on the braneworld models with scalar field [22-31]. Some of these studies are concentrated on the bulk scalar field minimally [23-25] or non-minimally[26-28] coupled to the bulk Ricci scalar. Some other authors have studied the minimally [29,30] or non-minimally [22,31] coupled scalar field to the induced Ricci scalar

on the brane. However, none of these studies have considered consequences of embedding of FRW cosmology with non-minimally coupled brane-scalar field into DGP scenario explicitly. Specifically, none of these studies obtained five dimensional metric components and full dynamics of the braneworld within this scenario trivially. In addition, there are very limited literature on late time behavior and quintessence with non-minimally coupled scalar field and even these studies have not performed explicit and detailed calculation of the late-time dynamics.

In this paper, in the spirit of DGP inspired gravity, we study the effect of an induced gravity term which is an arbitrary function of a scalar field on the brane. We present four-dimensional equations on a DGP brane with a scalar field non-minimally coupled to the induced Ricci curvature, embedded in a five-dimensional Minkowski bulk. This is an extension to a braneworld context of scalar-tensor (Brans-Dicke) gravity. We show that our model allows for an embedding of the standard Friedmann cosmology in the sense that the cosmological evolution of the background metric on the brane can entirely be described by the standard Friedmann equation plus energy non-conservation on the brane. In this framework we explore the relation between our formalism and the so-called dark radiation formalism. We study cosmological implications of both minimal and non-minimal extension of our model. In minimal case, motivated by modified theories of gravity, the potential describing minimally coupled scalar field is taken to be that with R^{-1} term added to the usual Einstein-Hilbert action[11]. In non-minimal case however, we concentrate mainly on the potential of the type $V(\phi) = \lambda\phi^n$. We study the weak field limit of our model and show that the mass density of ordinary matter on the brane should be modified by the addition of the effective mass density attributed to the non-minimally coupled scalar field on the brane. Considering the case of FRW brane, we obtain the evolution of the metric and scalar field by solving the field equations in the limit of small curvature. Our solutions for minimal case predict a power-law acceleration on the brane supporting observed late-time acceleration. For non-minimal case (by adapting a simple ansatz) we show that by a suitable choice of non-minimal coupling and scalar field potential one can achieve accelerated expansion in some special cases. We study quintessence model with non-minimally coupled scalar field on the brane and discuss some observational constraints imposed on the value of the non-minimal coupling using supernova data. We use a prime for differentiation with respect to fifth coordinate except for two special cases: $\alpha' \equiv \frac{d\alpha}{d\phi}$ and $V' \equiv \frac{dV}{d\phi}$. An overdot denotes differentiation with respect to the comoving time, t.

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2 Induced Gravity with Non-Minimally Coupled Brane-Scalar Field

The action of the DGP scenario in the presence of a non-minimally coupled scalar field on the brane can be written as follows

$$S = \int d^5x \frac{m_4^3}{2}\sqrt{-g}\mathcal{R} + \left[\int d^4x\sqrt{-q}\left(\frac{m_3^2}{2}\alpha(\phi)R[q] - \frac{1}{2}q^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi) + m_4^3\overline{K} + \mathcal{L}_m\right)\right]_{y=0},$$

(1)

where we have included a general non-minimal coupling $\alpha(\phi)$ in the brane part of the action(for an interesting discussion on the possible schemes to incorporate NMC in the formulation of scalar-tensor gravity see [22,35]). y is coordinate of the fifth dimension and we assume brane is located at $y = 0$. g_{AB} is five dimensional bulk metric with Ricci scalar $\mathcal{R}$, while $q_{\mu\nu}$ is induced metric on the brane with induced Ricci scalar R. g_{AB} and $q_{\mu\nu}$ are related via $q_{\mu\nu} = \delta_\mu{}^A\delta_\nu{}^B g_{AB}$. $\overline{K}$ is trace of the mean extrinsic curvature of the brane defined as

$$\overline{K}_{\mu\nu} = \frac{1}{2}\lim_{\epsilon\to 0}\left([K_{\mu\nu}]_{y=-\epsilon} + [K_{\mu\nu}]_{y=+\epsilon}\right),$$

(2)

and corresponding term in the action is York-Gibbons-Hawking term[32] (see also [20]). The ordinary matter part of the action is shown by Lagrangian $\mathcal{L}_m \equiv \mathcal{L}_m(q_{\mu\nu}, \psi)$ where ψ is matter field and corresponding energy-momentum tensor is

$$T_{\mu\nu} = -2\frac{\delta\mathcal{L}_m}{\delta q^{\mu\nu}} + q_{\mu\nu}\mathcal{L}_m.$$

(3)

The pure scalar field Lagrangian, $\mathcal{L}_\phi = -\frac{1}{2}q^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)$, yields the following energy-momentum tensor

$$\tau_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}q_{\mu\nu}(\nabla\phi)^2 - q_{\mu\nu}V(\phi)$$

(4)

The Bulk-brane Einstein's equations calculated from action (1) are given by

$$m_4^3\left(\mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R}\right) +$$

$$m_3^2\delta_A{}^\mu\delta_B{}^\nu\left[\alpha(\phi)\left(R_{\mu\nu} - \frac{1}{2}q_{\mu\nu}R\right) - \nabla_\mu\nabla_\nu\alpha(\phi) + q_{\mu\nu}\Box^{(4)}\alpha(\phi)\right]\delta(y) = \delta_A{}^\mu\delta_B{}^\nu\Upsilon_{\mu\nu}\delta(y),$$

(5)

where $\Box^{(4)}$ is 4-dimensional(brane) d'Alembertian and $\Upsilon_{\mu\nu} = T_{\mu\nu} + \tau_{\mu\nu}$. This relation can be rewritten as follows

$$m_4^3\left(\mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R}\right) + m_3^2\alpha(\phi)\delta_A{}^\mu\delta_B{}^\nu\left(R_{\mu\nu} - \frac{1}{2}q_{\mu\nu}R\right)\delta(y) = \delta_A{}^\mu\delta_B{}^\nu\mathcal{T}_{\mu\nu}\delta(y)$$

(6)

where $\mathcal{T}_{\mu\nu}$ is total energy-momentum on the brane defined as follows

$$\mathcal{T}_{\mu\nu} = m_3^2 \nabla_\mu \nabla_\nu \alpha(\phi) - m_3^2 q_{\mu\nu} \Box^{(4)} \alpha(\phi) + \Upsilon_{\mu\nu}, \tag{7}$$

From (6) we find

$$G_{AB} = \mathcal{R}_{AB} - \frac{1}{2} g_{AB} \mathcal{R} = 0 \tag{8}$$

and

$$G_{\mu\nu} = \left(R_{\mu\nu} - \frac{1}{2} q_{\mu\nu} R \right) = \frac{\mathcal{T}_{\mu\nu}}{m_3^2 \alpha(\phi)} \tag{9}$$

for bulk and brane respectively. The corresponding junction conditions relating the extrinsic curvature to the energy-momentum tensor of the brane, have the following form

$$\lim_{\epsilon \to +0} \left[K_{\mu\nu} \right]_{y=-\epsilon}^{y=+\epsilon} = \frac{1}{m_4^3} \left[\mathcal{T}_{\mu\nu} - \frac{1}{3} q_{\mu\nu} q^{\alpha\beta} \mathcal{T}_{\alpha\beta} \right]_{y=0} - \frac{m_3^2 \alpha(\phi)}{m_4^3} \left[R_{\mu\nu} - \frac{1}{6} q_{\mu\nu} q^{\alpha\beta} R_{\alpha\beta} \right]_{y=0}. \tag{10}$$

We set $g_{AB} = \eta_{AB} + h_{AB}$ where h_{AB} are small perturbations to investigate the weak field limit of the scenario. Within the Gaussian normal coordinate, we impose the following harmonic gauge on the longitudinal coordinates[9]

$$\partial_\alpha h^\alpha{}_\mu + \partial_y h_{y\mu} = \frac{1}{2} \partial_\mu \left(h^\alpha{}_\alpha + h_{yy} \right). \tag{11}$$

This will led us to a decoupled equation for the gravitational potential of a static mass distribution. The transverse equations in the gauge (11) give $h_{y\mu} = 0$, $h_{yy} = h^\alpha{}_\alpha$. The remaining equations take the following form

$$m_4^3 (\partial_\alpha \partial^\alpha + \partial_y^2) h_{\mu\nu} + m_3^2 \alpha(\phi) \left(\partial_\alpha \partial^\alpha h_{\mu\nu} - \partial_\mu \partial_\nu h_\alpha{}^\alpha \right) \delta(y) = -2\delta(y) \left[\mathcal{T}_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} \eta^{\alpha\beta} \mathcal{T}_{\alpha\beta} \right]. \tag{12}$$

We suppose that non-minimally coupled scalar field has an effective mass M_ϕ. The gravitational potential of mass densities $\rho_\psi(\vec{r}) = M_\psi \delta(\vec{r})$ and $\rho_\phi(\vec{r}) = M_\phi \delta(\vec{r})$ on the brane satisfies the following equation

$$m_4^3 \left(\partial_\alpha \partial^\alpha + \partial_y^2 \right) U(\vec{r}, y) + m_3^2 \alpha(\phi) \delta(y) \partial_\alpha \partial^\alpha U(\vec{r}, y) = \frac{2}{3} (M_\psi + M_\phi) \delta(\vec{r}) \delta(y). \tag{13}$$

This equation shows that in the presence of non-minimally coupled scalar field, the mass in standard DGP framework should be modified by the addition of the mass of the non-minimally coupled scalar field. Using the following Fourier ansatz

$$U(\vec{r}, y) = \frac{1}{(2\pi)^4} \int d^3\vec{p} \int dp_y \, U(\vec{p}, p_y) \exp \left[i(\vec{p} \cdot \vec{r} + p_y y) \right] \tag{14}$$

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in equation (13), we find

$$m_4^3(\vec{p}^2 + p_y^2)U(\vec{p}, p_y) + \frac{m_3^2\alpha(\phi)}{2\pi}\vec{p}^2\int dp_y'\, U(\vec{p}, p_y') = -\frac{2}{3}(M_\psi + M_\phi). \qquad (15)$$

This integral equation has the following solution

$$U(\vec{p}, p_y) = -\frac{4}{3}\frac{M_\psi + M_\phi}{(\vec{p}^2 + p_y^2)\big(2m_4^3 + m_3^2\alpha(\phi)|\vec{p}|\big)}. \qquad (16)$$

The resulting potential on the brane is

$$U(\vec{r}) = -\left(\frac{M_\psi + M_\phi}{6\pi m_3^2\alpha(\phi)r}\right)\left[\cos(\xi_\alpha r) - \frac{2}{\pi}\cos(\xi_\alpha r)\,\mathrm{Si}(\xi_\alpha r) + \frac{2}{\pi}\sin(\xi_\alpha r)\,\mathrm{Ci}(\xi_\alpha r)\right], \qquad (17)$$

where $\xi_\alpha = \frac{2m_4^3}{m_3^2\alpha(\phi)}$ and the *sine* and *cosine* integrals are defined as follows

$$\mathrm{Si}(x) = \int_0^x d\omega\,\frac{\sin\omega}{\omega}, \qquad\qquad \mathrm{Ci}(x) = -\int_x^\infty d\omega\,\frac{\cos\omega}{\omega}.$$

Now there is a modified transition scale

$$\xi_\alpha^{-1} \equiv \ell_\alpha = \frac{m_3^2\alpha(\phi)}{2m_4^3} = \alpha(\phi)\ell_{DGP} \qquad (18)$$

between four and five-dimensional behavior of the gravitational potential in this scenario:

$$r \ll \ell_\alpha: \qquad U(\vec{r}) = -\frac{M_\psi + M_\phi}{6\pi m_3^2\alpha(\phi)r}\left[1 + \left(\gamma - \frac{2}{\pi}\right)\frac{r}{\ell_\alpha} + \frac{r}{\ell_\alpha}\ln\left(\frac{r}{\ell_\alpha}\right) + \mathcal{O}\left(\frac{r^2}{\ell_\alpha^2}\right)\right], \qquad (19)$$

and

$$r \gg \ell_\alpha: \qquad U(\vec{r}) = -\frac{M_\psi + M_\phi}{6\pi^2 m_4^3 r^2}\left[1 - 2\frac{\ell_\alpha^2}{r^2} + \mathcal{O}\left(\frac{\ell_\alpha^4}{r^4}\right)\right], \qquad (20)$$

where $\gamma = 0.577$ is Euler's constant. Therefore, existence of non-minimally coupled scalar field on the brane has the following consequences: the mass density of ordinary matter on the brane should be modified by the addition of the mass density attributed to the scalar field on the brane and the DGP transition scale between four and five dimensional behavior of gravitational potential now is explicitly dependent on the strength of non-minimal coupling. If $\alpha(\phi)$ varies slightly from point to point on the brane, it can be interpreted as a spacetime dependent Newton's constant. The dynamics that control this variation are determined by the following equation

$$\nabla^\mu\phi\nabla_\mu\phi - \frac{dV}{d\phi} + \frac{m_3^2}{2}\left(\frac{d\alpha(\phi)}{d\phi}\right)R = 0. \qquad (21)$$

Note that in the real world we don't want $\alpha(\phi)$ to vary too much since it will have observable effects in classic experimental tests of general relativity and also in cosmological tests such as primordial nucleosynthesis[33]. This can be ensured either by choosing a large mass for scalar field ϕ or choosing $\alpha(\phi)$ so that large changes in ϕ give rise to relatively small changes in Newton's constant. So, when $\alpha(\phi)$ varies in DGP brane from point to point, the crossover scale will change and is no longer a constant. This feature would change previous picture of crossover scale in DGP scenario and my change some arguments on phenomenology of this scenario[34]. Note that we can define a modified four-dimensional Planck mass as $m_3^{(\alpha)} = \sqrt{\alpha(\phi)} m_3$ and therefore the effect of non-minimal coupling can be attributed to the modification of four dimensional Planck mass. This is equivalent to modification of four dimensional Newton's constant[22]. If we use the reduced Planck mass for m_3, gravitational potential for small r limit will differ from the ordinary four-dimensional potential by a factor $\frac{4}{3}\alpha^{-1}$. In fact the coupling of the masses on the brane to the induced Ricci tensor on the brane is modified by this factor. Existence of this extra factor is in agreement with the tensorial structure of the graviton propagator due to additional helicity state of the five-dimensional graviton [9,34]. Figure 1 shows the shape of the DGP potential for $r \ll \ell_\alpha$ (equation (19)) with some arbitrary values of the NMC. As this figure shows, for large negative values of NMC, it is possible to have repulsive gravitational potential. For positive values of NMC the DGP potential for $r \ll \ell_\alpha$ is always attractive. Figure 2 shows the DGP potential for $r \gg \ell_\alpha$ (equation (20)).
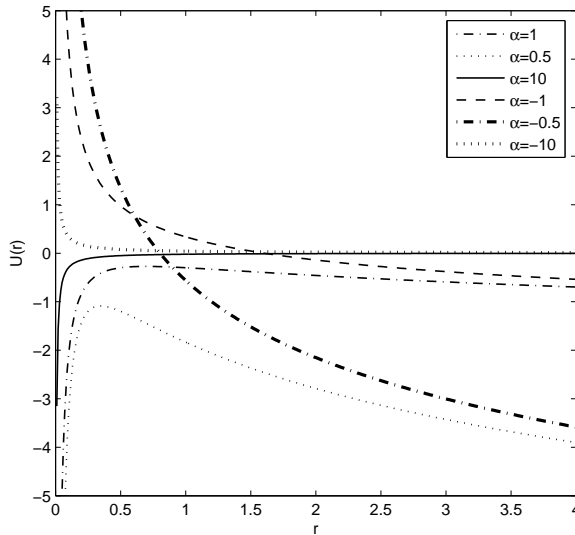

Figure 1: DGP potential for $r \ll \ell_\alpha$ and with some arbitrary values of NMC (equation (19)).

Ignoring the tensor structure, the gravitational potential of a source of mass m is given
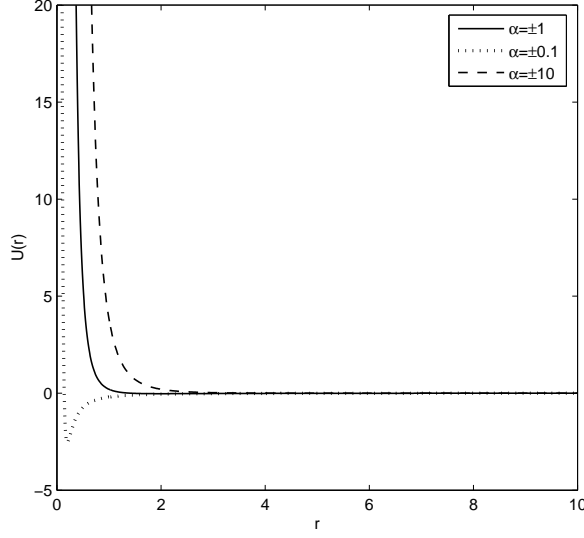
Figure 2: DGP potential for $r \gg \ell_\alpha$ and with some arbitrary values of NMC (equation (20)).

by $U_{grav} \sim -\frac{G_{brane}m}{r}$ for $r \ll \ell_\alpha$ and $U_{grav} \sim -\frac{G_{bulk}m}{r^2}$ for $r \gg \ell_\alpha$ respectively. That is, the potential exhibits four-dimensional behavior at short distances and five-dimensional behavior (i.e., as if the brane were not there at all) at large distances. In the absence of NMC, for the crossover scale to be large, we need a substantial mismatch between four-dimensional Planck scale (corresponding to the usual Newton's constant, $G_{brane} = G$) and the fundamental, or bulk, Planck scale M_4 [34]. However, with NMC $\alpha(\phi)$, evidently ℓ_α can be large with large values of $\alpha(\phi)$. Therefore existence of NMC changes the mismatch condition between four-dimensional Planck scale and the fundamental Planck scale. In minimal case, the fundamental Planck scale M_4 has to be quite small in order for the energy of gravity fluctuations to be substantially smaller in the bulk versus on the brane, the energy of the latter being controlled by $M_3 = M_P$. This situation can be mediate due to the presence of NMC. Note that when M_4 is small, the corresponding Newton's constant in the bulk, G_{bulk}, is large. So, for a given source mass m, gravity is much stronger in the bulk. Earlier work using Supernova data implies that the best-fit for density parameter of ordinary matter is given by $\Omega_M^0 = 0.18^{+0.07}_{-0.06}$ [41,42]. We may introduce a new effective dark energy component, Ω_{ℓ_α}, where $\Omega_{\ell_\alpha} = \frac{1}{\ell_\alpha H}$ to resort the identity: $1 = \Omega_M + \Omega_{\ell_\alpha}$. Therefore we find $\ell_\alpha = \left(1.21^{+0.09}_{-0.09}\right)H_0^{-1}$. Assuming a flat universe, a more recent supernova data [43] suggests a best-fit of $\Omega_M = 0.21$ corresponding to a best-fit crossover scale $\ell_\alpha = 1.26H_0^{-1}$. Since $\ell_\alpha = \frac{m_3^2 \alpha(\phi)}{2m_4^3}$ or $\alpha(\phi) = \frac{2m_4^3 \ell_\alpha}{m_3^2}$, for $M_3 \sim 10^{18}GeV$ and $M_4 \sim 10^3 GeV$, we find $\alpha(\phi) \sim 2.52 \times 10^{-30} H_0^{-1}$.

3 Non-Minimal DGP Cosmology

It has been shown that DGP model can account for the standard Friedmann cosmology at any distance scale on the brane [20]. In our proposed framework, we start with the following line element to derive cosmological implications of our model,

$$ds^2 = q_{\mu\nu}dx^\mu dx^\nu + b^2(y,t)dy^2 = -n^2(y,t)dt^2 + a^2(y,t)\gamma_{ij}dx^i dx^j + b^2(y,t)dy^2. \quad (22)$$

In this relation γ_{ij} is a maximally symmetric 3-dimensional metric defined as

$$\gamma_{ij} = \delta_{ij} + k\frac{x_i x_j}{1 - kr^2} \quad (23)$$

where $k = -1, 0, 1$ parameterizes the spatial curvature and $r^2 = x_i x^i$. We assume that scalar field ϕ depends only on the proper cosmic time of the brane. Choosing gauge $b^2(y,t) = 1$ in Gaussian normal coordinates, the field equations in the bulk are given by (8) with the following Einstein's tensor components

$$G_{00} = 3n^2\Big(\frac{\dot{a}^2}{n^2 a^2} - \frac{a'^2}{a^2} - \frac{a''}{a} + \frac{k}{a^2}\Big), \quad (24)$$

$$G_{ij} = \gamma_{ij}a^2\Big[\Big(\frac{a'^2}{a^2} - \frac{\dot{a}^2}{n^2 a^2} - \frac{k}{a^2}\Big) + 2\Big(\frac{a''}{a} + \frac{n'a'}{na} - \frac{\ddot{a}}{n^2 a} + \frac{\dot{n}\dot{a}}{n^3 a} + \frac{n''}{2n}\Big)\Big] \quad (25)$$

$$G_{0y} = 3\Big(\frac{n'}{n}\frac{\dot{a}}{a} - \frac{\dot{a}'}{a}\Big), \quad (26)$$

$$G_{yy} = 3\Big(\frac{a'^2}{a^2} - \frac{\dot{a}^2}{n^2 a^2} - \frac{k}{a^2} + \frac{n'a'}{na} + \frac{\dot{n}\dot{a}}{n^3 a} - \frac{\ddot{a}}{n^2 a}\Big). \quad (27)$$

The field equations on the brane are given by the following equations

$$G_{00}^{(3)} = 3n^2\left(\frac{\dot{a}^2}{n^2 a^2} + \frac{k}{a^2}\right) = \frac{2}{m_3^2 \alpha(\phi)}\mathcal{T}_{00}, \quad (28)$$

$$G_{ij}^{(3)} = \gamma_{ij}\Big[2\Big(\frac{\dot{n}\dot{a}}{n^3 a} - \frac{\ddot{a}}{n^2 a}\Big) - \Big(\frac{\dot{a}^2}{n^2 a^2} + \frac{k}{a^2}\Big)\Big] = \frac{2}{m_3^2 \alpha(\phi)}\mathcal{T}_{ij}, \quad (29)$$

and scalar field evolution equation

$$\ddot{\phi} + \Big(3\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\Big)\dot{\phi} + n^2\frac{dV}{d\phi} - \frac{m_3^2}{2}n^2\alpha' R[q] = 0, \quad (30)$$

where Ricci scalar on the brane is given by

$$R = 3\frac{k}{a^2} + \frac{1}{n^2}\Big[6\frac{\ddot{a}}{a} + 6\Big(\frac{\dot{a}}{a}\Big)^2 - 6\frac{\dot{a}}{a}\frac{\dot{n}}{n}\Big]. \quad (31)$$

The other important equation is the continuity equation on the brane. Suppose that ordinary matter on the brane has an ideal fluid form, $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pq_{\mu\nu}$. Since $K_{tt} = nn'$ and $K_{rr} = -aa'$, equation (10) gives the following matching conditions

$$\lim_{\epsilon \to +0} [\partial_y a]_{y=-\epsilon}^{y=+\epsilon}(t) = \frac{m_3^2}{m_4^3}\left[\alpha(\phi)\left(\frac{\dot{a}^2}{n^2 a} + \frac{k}{a}\right)\right]_{y=0} - \left[\frac{(\rho + \rho_\phi)a}{3m_4^3}\right]_{y=0}. \tag{32}$$

$$\lim_{\epsilon \to +0} [\partial_y n]_{y=-\epsilon}^{y=+\epsilon}(t) = \frac{m_3^2}{m_4^3}(2n)\left[\alpha(\phi)\left(\frac{\ddot{a}}{n^2 a} - \frac{\dot{a}^2}{2n^2 a^2} - \frac{\dot{n}\dot{a}}{n^3 a} - \frac{k}{2a^2}\right)\right]_{y=0} +$$
$$\frac{n}{3m_4^3}\left[2(\rho + \rho_\phi) + 3(p + p_\phi)\right]_{y=0} \tag{33}$$

where energy density and pressure of non-minimally coupled scalar field are given as follows

$$\rho_\phi = \left[\frac{1}{2}\dot{\phi}^2 + n^2 V(\phi) - 6\alpha' H \dot{\phi}\right]_{y=0}, \tag{34}$$

$$p_\phi = \left[\frac{1}{2n^2}\dot{\phi}^2 - V(\phi) + \frac{2\alpha'}{n^2}\left(\ddot{\phi} - \frac{\dot{n}}{n}\dot{\phi}\right) + 4\alpha'\frac{H}{n^2}\dot{\phi} + \frac{2\alpha''}{n^2}\dot{\phi}^2\right]_{y=0}, \tag{35}$$

and $H = \frac{\dot{a}}{a}$ is Hubble parameter. Note that part of the effect of non-minimal coupling of the field ϕ is hidden in the definition of the effective energy density and pressure which both include non-minimal terms. Now using (26), since in the bulk $G_{00} = 0$, we find

$$\lim_{\epsilon \to +0}\left[\frac{n'}{n}\right]_{y=-\epsilon}^{y=+\epsilon} = \left[\frac{\dot{a}'}{\dot{a}}\right]_{y=-\epsilon}^{y=+\epsilon} \tag{36}$$

using relations (32) and (33) we find the following relation for conservation of energy on the brane

$$\dot{\rho} + \dot{\rho}_\phi + 3H\left(\rho + \rho_\phi + p + p_\phi\right) = 6\alpha'\dot{\phi}\left(H^2 + \frac{k}{a^2}\right). \tag{37}$$

Thus the non-minimal coupling of the scalar field to the Ricci curvature on the brane through $\alpha(\phi)$ leads to the non-conservation of the effective energy density.

To obtain the cosmological dynamics, we set $n(0, t) = 1$. With this gauge condition we recover usual time on the brane via transformation $t = \int^t n(0, \eta)d\eta$. In this situation, our basic dynamical variable is only $a(y, t)$ since $n(y, t)$ now is given by

$$n(y, t) = \frac{\dot{a}(y, t)}{\dot{a}(0, t)}. \tag{38}$$

where $H = \frac{\dot{a}(0,t)}{a(0,t)}$ is Hubble parameter on the brane. Now we can write the basic set of cosmological equations for a FRW brane in the presence of a non-minimally coupled scalar field. The first of these equations is given by matching condition

$$\lim_{\epsilon \to +0} [\partial_y a]_{y=-\epsilon}^{y=+\epsilon}(t) = \frac{m_3^2}{m_4^3}\left[\alpha(\phi)\left(\frac{\dot{a}^2}{n^2 a} + \frac{k}{a}\right)\right]_{y=0} - \left[\frac{(\rho + \rho_\phi)a}{3m_4^3}\right]_{y=0}. \tag{39}$$

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Insertion of $\frac{n'}{n} = \frac{\dot{a}'}{\dot{a}}$ into equations (24) and (27) yields

$$\frac{2}{3n^2} a' a^3 G_{00} = \frac{\partial}{\partial y}\left(\frac{\dot{a}^2}{n^2}a^2 - a'^2 a^2 + ka^2\right) = 0$$

and

$$\frac{2}{3}\dot{a}a^3 G_{yy} = -\frac{\partial}{\partial t}\left(\frac{\dot{a}^2}{n^2}a^2 - a'^2 a^2 + ka^2\right) = 0.$$

These two equations imply that the Binétruy *et al* [14] integrals

$$\mathcal{I}^+ = \left[\left(\frac{\dot{a}^2}{n^2} - a'^2 + k\right)a^2\right]_{y>0}, \tag{40}$$

and

$$\mathcal{I}^- = \left[\left(\frac{\dot{a}^2}{n^2} - a'^2 + k\right)a^2\right]_{y<0}, \tag{41}$$

are constant and if a' is continuous on the brane then $\mathcal{I}^+ = \mathcal{I}^-$. These equations along with scalar field equation

$$\ddot{\phi} + \left(3\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right)\dot{\phi} + n^2\frac{dV}{d\phi} - n^2\frac{d\alpha}{d\phi}R[q] = 0, \tag{42}$$

and

$$n(y,t) = \frac{\dot{a}(y,t)}{\dot{a}(0,t)}. \tag{43}$$

constitute the basic dynamical equations of our model. In the absence of transverse momentum, $\Upsilon_{0y} = 0$, one can show that $\mathcal{I}^+ = \mathcal{I}^-$. In fact $\mathcal{I}^\pm$ can be considered as initial conditions and these quantities reflect the symmetry across the brane. In the case of $\mathcal{I}^+ \neq \mathcal{I}^-$ there can not be any symmetry across the brane. So we first consider the case $\mathcal{I}^+ = \mathcal{I}^-$ in which follows. Our cosmological equations on the brane now take the following forms(note that $n(0,t) = 1$)

$$\frac{\dot{a}^2(0,t) + k}{a^2(0,t)} = \frac{(\rho + \rho_\phi)}{3m_3^2\alpha(\phi)}, \tag{44}$$

$$\ddot{\phi} + 3\frac{\dot{a}(0,t)}{a(0,t)}\dot{\phi} + \frac{dV(\phi)}{d\phi} = \frac{d\alpha}{d\phi}R[q], \tag{45}$$

$$\mathcal{I} = \left[\dot{a}^2(0,t) - a'^2(y,t) + k\right]a^2(y,t) \tag{46}$$

$$n(y,t) = \frac{\dot{a}(y,t)}{\dot{a}(0,t)}. \tag{47}$$

Using equation (46), the scale factor is calculated as follows

$$a^2(y,t) = a^2(0,t) + \left[\dot{a}^2(0,t) + k\right]y^2 + 2\left[\left(\dot{a}^2(0,t) + k\right)a^2(0,t) - \mathcal{I}\right]^{\frac{1}{2}}y \tag{48}$$

and therefore $n(y,t)$ is given by equation (47);

$$n(y,t) = \left(a(0,t) + \ddot{a}(0,t)y^2 + a(0,t)\frac{a(0,t)\ddot{a}(0,t) + \dot{a}^2(0,t) + k}{\sqrt{\left(\dot{a}^2(0,t) + k\right)a^2(0,t) - \mathcal{I}}}y \right)$$

$$\times \left[a^2(0,t) + \left[\dot{a}^2(0,t) + k\right]y^2 + 2\left[\left(\dot{a}^2(0,t) + k\right)a^2(0,t) - \mathcal{I}\right]^{\frac{1}{2}}y \right]^{\frac{-1}{2}} \tag{49}$$

So, the components of 5-dimensional metric (22) are determined. If we set initial conditions in such a way that $\mathcal{I} = 0$, we find the following simple equations for cosmological dynamics

$$a(y,t) = a(0,t) + \left[\dot{a}^2(0,t) + k\right]^{\frac{1}{2}}y, \tag{50}$$

$$n(y,t) = 1 + \frac{\ddot{a}(0,t)}{\sqrt{\dot{a}^2(0,t) + k}}y. \tag{51}$$

Therefore, our model allows for an embedding of the standard Friedmann cosmology in the sense that the cosmological evolution of the background metric on the brane can be described by the standard Friedmann equation plus energy non-conservation on the brane.

So far we have discussed the case $\mathcal{I}^+ = \mathcal{I}^-$ with a continuous warp factor across the brane. In the case of $\mathcal{I}^+ \neq \mathcal{I}^-$, there cannot be any symmetry across the brane. In this case the basic set of dynamical equations is provided by equations (39), (40) and (41) plus the non-conservation of the effective energy density given by (37). In this case, evolution of the scale factor on the brane is given by elimination of $a'(y \longrightarrow \pm 0, t)$ from the following generalized Friedmann equation

$$\pm\left[\dot{a}^2(0,t) + k - a^{-2}(0,t)\mathcal{I}^+\right]^{\frac{1}{2}} \mp \left[\dot{a}^2(0,t) + k - a^{-2}(0,t)\mathcal{I}^-\right]^{\frac{1}{2}}$$

$$= \alpha(\phi)\frac{m_3^2}{m_4^3}\left(\frac{\dot{a}^2(0,t) + k}{a(0,t)}\right) - \frac{(\rho + \rho_\phi)a(0,t)}{3m_4^3}. \tag{52}$$

This is the most general form of the modified Friedmann equation for our non-minimal framework. After determination of $a(0,t)$, since $\mathcal{I}^\pm$ are constants, $a(y,t)$ can be calculated from (46). This is the full dynamics of the system. Note that in the case where the right hand side of equation (52) is negative, at least one sign in the left hand side should be negative depending on initial conditions. However, the dynamics of the problem does not require symmetry across the brane. Therefore, we have shown the possibility of embedding of FRW cosmology in DGP scenario with a 4D non-minimally coupled scalar field on the brane and equation (52) is the most general form of FRW equation in this embedding.

4 Dark Radiation Formalism with Non-Minimal Coupling

In order to show that our model is consistent with dark energy formulation of Friedmann equation, we first obtain non-minimal extension of this formalism and then we show that this equation can be obtained in our framework more easily. The effective Einstein equation on the brane is given by[24]

$$G_{\mu\nu} = \frac{\Pi_{\mu\nu}}{m_4^6} - \mathcal{E}_{\mu\nu}, \tag{53}$$

where

$$\Pi_{\mu\nu} = -\frac{1}{4}\mathcal{T}_{\mu\sigma}\mathcal{T}_{\nu}{}^{\sigma} + \frac{1}{12}\mathcal{T}\mathcal{T}_{\mu\nu} + \frac{1}{8}g_{\mu\nu}\Big(\mathcal{T}_{\rho\sigma}\mathcal{T}^{\rho\sigma} - \frac{1}{3}\mathcal{T}^2\Big), \tag{54}$$

and $\mathcal{T}_{\mu\nu}$ is given by equation (7). Also we have

$$\mathcal{E}_{\mu\nu} = C_{MRNS}\, n^M\, n^N g^R{}_\mu\, g^S{}_\nu \tag{55}$$

where C_{MRNS} is five dimensional Weyl tensor and n_A is the spacelike unit vector normal to the brane. Now, using equation (53) we find

$$G^0{}_0 = \frac{\Pi^0{}_0}{m_4^6} - \mathcal{E}^0{}_0 \tag{56}$$

where for FRW universe we have

$$G^0{}_0 = -3\Big(H^2 + \frac{k}{a^2}\Big). \tag{57}$$

Similarly, for space components we have

$$G^i{}_j = \frac{\Pi^i{}_j}{m_4^6} - \mathcal{E}^i{}_j \tag{58}$$

where

$$G^i{}_j = -\Big(2\dot{H} + 3H^2 + \frac{k}{a^2}\Big)\delta^i{}_j. \tag{59}$$

Now using equation (54) we find $\Pi^0{}_0 = -\frac{1}{12}\big(\mathcal{T}^0{}_0\big)^2$ and $\Pi^i{}_j = -\frac{1}{12}\mathcal{T}^0{}_0\big(\mathcal{T}^0{}_0 - 2\mathcal{T}^1{}_1\big)\delta^i{}_j$. Also, equation (7) gives

$$\mathcal{T}^0{}_0 = -(\rho + \rho_\phi) - m_3^2\alpha(\phi)G^0{}_0 \tag{60}$$

and

$$\mathcal{T}^i{}_j = -(p + p_\phi)\delta^i{}_j - m_3^2\alpha(\phi)G^i{}_j, \tag{61}$$

where ρ_ϕ and p_ϕ are given by (34) and (35) with $n(0,t) = 1$. These equations lead us to the following generalized Friedmann equation

$$3\left(H^2 + \frac{k}{a^2}\right) = \mathcal{E}^0{}_0 + \frac{1}{12m_4^6}\left[\rho + \rho_\phi - 3m_3^2\alpha(\phi)\left(H^2 + \frac{k}{a^2}\right)\right]^2. \tag{62}$$

Using Codazzi equation we have $\nabla^\nu \mathcal{E}_{\mu\nu} = 0$ (see for example [24,31]). Therefore we find $\dot{\mathcal{E}}^0{}_0 + 4H\mathcal{E}^0{}_0 = 0$ which integration gives $\mathcal{E}^0{}_0 = \frac{\mathcal{E}_0}{a^4}$ with $\mathcal{E}_0$ as an integration constant. Therefore, equation (62) can be re-written as follows

$$H^2 + \frac{k}{a^2} = \frac{1}{3m_3^2\alpha(\phi)}\left(\rho + \rho_\phi + \rho_0\left[1 + \varepsilon\sqrt{1 + \frac{2}{\rho_0}\left[\rho + \rho_\phi - m_3^2\alpha(\phi)\frac{\mathcal{E}_0}{a^4}\right]}\,\right]\right). \tag{63}$$

where $\rho_0 = \frac{6m_4^6}{m_3^2\alpha(\phi)}$ and $\varepsilon = \pm 1$ shows the possibility of existence of two different branches of FRW equation. In the high energy regime where $\frac{\rho+\rho_\phi}{\rho_0} \gg 1$, we find

$$H^2 \approx \frac{1}{3m_3^2\alpha(\phi)}\left(\rho + \rho_\phi + \varepsilon\sqrt{2(\rho + \rho_\phi)\rho_0}\,\right) \tag{64}$$

which describes a four dimensional gravity with a small correction. In the low energy regime where $\frac{\rho+\rho_\phi}{\rho_0} \ll 1$, we find

$$H^2 \approx \frac{1}{3m_3^2\alpha(\phi)}\left[(1 + \varepsilon)(\rho + \rho_\phi) + (1 + \varepsilon)\rho_0 - \frac{\varepsilon}{4}\frac{(\rho + \rho_\phi)^2}{\rho_0}\right]. \tag{65}$$

Now we show that this result can be obtained in our formalism more easily. For this purpose we show that relation (52) for the case with $\mathcal{I}^+ = \mathcal{I}^- \equiv \mathcal{I}$ and a discontinuous warp factor across the Z_2 symmetric brane leads to this result with some simple algebra. For simplicity, we define $x \equiv H^2 + \frac{k}{a^2}$, $b \equiv \rho + \rho_\phi$, $y \equiv \alpha(\phi)m_3^2$, and $z \equiv m_4^3$. With these definitions, equation (52) (with upper sign for instance), transforms to the following form

$$\left(x - \frac{\mathcal{I}^+}{a^4}\right)^{\frac{1}{2}} + \left(x - \frac{\mathcal{I}^-}{a^4}\right)^{\frac{1}{2}} = \frac{y}{z}x - \frac{b}{3z}. \tag{66}$$

Solving this equation for x (with $\mathcal{I}^+ = \mathcal{I}^- \equiv \mathcal{I}$) gives the following result

$$x = \frac{\frac{by}{3z^2} + 2 \pm \sqrt{\left(\frac{by}{3z^2} + 2\right)^2 - \frac{y^2}{z^2}\left(\frac{b^2}{9z^2} + \frac{4\mathcal{I}}{a^4}\right)}}{\frac{y^2}{z^2}}. \tag{67}$$

A little algebraic manipulation gives

$$x = \frac{1}{3y}\left[b + \frac{6z^2}{y} \pm \frac{6z^2}{y}\sqrt{1 + \frac{by}{3z^2} - \frac{\mathcal{I}y^2}{a^4z^2}}\right]. \tag{68}$$

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Considering both plus and minus signs in equation (52) and using original quantities we obtain

$$H^2 + \frac{k}{a^2} = \frac{1}{3m_3^2\alpha(\phi)}\left(\rho + \rho_\phi + \rho_0\left[1 + \varepsilon\sqrt{1 + \frac{2}{\rho_0}\left[\rho + \rho_\phi - m_3^2\alpha(\phi)\frac{\mathcal{E}_0}{a^4}\right]}\right]\right). \tag{69}$$

where $\rho_0 \equiv \frac{6z^2}{y} = \frac{6m_4^6}{m_3^2\alpha(\phi)}$ and $\mathcal{E}_0 = 3\mathcal{I}$ is a constant. This analysis shows the consistency of our formalism with dark-radiation formalism presented above.

5 Cosmological Considerations

Now to discover cosmological implications of non-minimally coupled scalar field on DGP braneworld, we proceed as follows: the evolution of scalar field on the brane for spatially flat FRW metric ($k = 0$) is given by the following equations

$$H_0^2 = \frac{1}{3\alpha(\phi)m_3^2}(\rho + \rho_\phi), \tag{70}$$

and

$$\ddot{\phi} + 3H_0\dot{\phi} + \frac{dV(\phi)}{d\phi} = \alpha'R[q], \tag{71}$$

where $H_0 \equiv \frac{\dot{a}(0,t)}{a(0,t)}$ and for $n(0,t) = 1$ (on the brane) we have

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\alpha'H\dot{\phi}. \tag{72}$$

Ricci scalar on the brane is given by the following relation

$$R = 6\frac{\ddot{a}}{a} + 6\left(\frac{\dot{a}}{a}\right)^2. \tag{73}$$

In the absence of ordinary matter on the brane we set $\rho = 0$. To find cosmological implications of our model we should solve equations (70) and (71) using (72) and (73). For comparison purpose, we first give an overview of the minimal case, that is, the case with $\alpha = constant$ which has been discussed with details in reference [30]. First we should specify the form of the scalar field potential $V(\phi)$. We use the following potential which has been motivated from theories of modified gravity with Lagrangian of the type $\mathcal{L}(R) = R - \frac{\mu^4}{R}$ with R^{-1} term[11,30]

$$V(\phi) \simeq \mu^2 m_3^2 \exp\left(-\sqrt{\frac{3}{2}}\frac{\phi}{m_3}\right). \tag{74}$$

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In this case we find $a(0,t) \propto t^{4/3}$ and $\phi \propto -\frac{4}{3}\ln t$ for the evolution of scale factor and scalar field on the brane respectively. Obviously, this predicts a power-law acceleration on the brane. This result is consistent with the observational results similar to quintessence with the equation of state parameter $-1 < w_{DE} < -\frac{1}{3}$, [6,20]. In this case, using (48) and (49), the evolution of $a(y,t)$ and $n(y,t)$ is given by the following equations [30]

$$a^2(y,t) = C^2\left(t^{\frac{8}{3}} + \frac{16}{9}t^{\frac{2}{3}}y^2\right) + 2\left(\frac{16}{9}C^4 t^{\frac{10}{3}} - \mathcal{I}\right)^{\frac{1}{2}} y \tag{75}$$

and

$$n(y,t) = C\left[t^{\frac{4}{3}} + \frac{4}{9}t^{-\frac{2}{3}}y^2 + \frac{20}{9}C^2 t^{\frac{8}{3}}\left(\frac{16}{9}C^4 t^{\frac{10}{3}} - \mathcal{I}\right)^{\frac{-1}{2}} y\right]\frac{1}{a(y,t)}, \tag{76}$$

where C is a constant and $\mathcal{I} = \left[\dot{a}^2(0,t) - a'^2(y,t) + k\right]a^2$. If we set the initial conditions in such a way that $\mathcal{I} = 0$, these results become very simple

$$a(y,t) = C\left(t^{\frac{4}{3}} + \frac{4}{3}t^{\frac{1}{3}}y\right), \qquad n(y,t) = C\left(1 + \frac{y}{3t}\right). \tag{77}$$

In the case where $\alpha(\phi)$ is not just a constant, the situation becomes more complicated since now the kind of the cosmological solutions depend explicitly on the value of non-minimal coupling. We first try to obtain a necessary condition for the acceleration of the universe in an specific model. Suppose that $\rho = 0$. By definition, the required condition for acceleration of the universe is $\rho_\phi + 3p_\phi < 0$. Using equations defining ρ_ϕ and p_ϕ, we find

$$(1 + 3\alpha'')\dot{\phi}^2 - V(\phi) + 3\alpha'(H\dot{\phi} + \ddot{\phi}) < 0. \tag{78}$$

Using Klein-Gordon equation (71), this relation can be rewritten as follows

$$(1 + 3\alpha'')\dot{\phi}^2 - V(\phi) + 3\alpha'^2 R - 6\alpha' H_0 \dot{\phi} - 3\alpha'\frac{dV}{d\phi} < 0. \tag{79}$$

Finally, using (72), this can be written as

$$\rho_\phi - 2V(\phi) + (\frac{1}{2} + 3\alpha'')\dot{\phi}^2 + 3\alpha'^2 R - 3\alpha'\frac{dV}{d\phi} < 0. \tag{80}$$

This is a general condition to have an accelerating universe with non-minimally coupled scalar field on the brane [22,35]. To proceed further, we assume weak energy condition $\rho_\phi \geq 0$. Motivated from several theoretical evidences(for example: conformal coupling in general relativity and other metric theories[35], renormalization group study of $\lambda\phi^4$ theory[36] and large N limit of the Nambu-Jona-Lasinio model[37], (see also [22] and references therein), in which follows we set $\alpha(\phi) = \frac{1}{2}(1 - \xi\phi^2)$ with $\xi \leq \frac{1}{6}$. Therefore we find

$$V - \frac{3\xi}{2}\phi\frac{dV}{d\phi} > 0. \tag{81}$$

As an example, suppose that $V(\phi) = \lambda \phi^n$. In this case with $\xi \leq \frac{1}{6}$ and using (81), we find

$$\lambda\left(1 - \frac{3n\xi}{2}\right) > 0. \tag{82}$$

If we assume a positive scalar field potential with $\lambda > 0$, the condition for accelerating expansion restricts ξ to the values which $\xi \leq \frac{2}{3n}$. So in the presence of non-minimally coupled scalar field, it is harder to achieve accelerating universe with usual potentials.

Now with above definitions of non-minimal coupling and scalar field potential, equations (70) and (71) take the following forms

$$\frac{\dot{a}^2}{a^2} = \frac{2}{3}(1 - \xi\phi^2)^{-1}\left[\frac{1}{2}\dot{\phi}^2 + \lambda\phi^n + 6\xi\phi\frac{\dot{a}}{a}\dot{\phi}\right], \tag{83}$$

and

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + n\lambda\phi^{n-1} + 6\xi\phi\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right] = 0. \tag{84}$$

In order to study late time behavior of these equations, we try the following ansatz,

$$a(t) \approx A\,t^\nu, \qquad \phi(t) \approx B\,t^{-\mu} \tag{85}$$

where we have assumed a decreasing power law ansatz for scalar field. With these choices and setting $n = 4$, equation (83) gives

$$\frac{3}{2}\frac{\nu^2}{t^2} - \frac{3}{2}\xi B^2\nu^2 t^{-2\mu-2} = \left(\frac{1}{2}B^2\mu^2 - 6\xi B^2\mu\nu\right)t^{-2\mu-2} + \lambda B^4 t^{-4\mu} \tag{86}$$

On the other hand, equation (84) gives

$$\left[\mu(\mu + 1) - 3\mu\nu + 6\xi(2\nu^2 + \nu)\right]t^{-\mu-2} + 4\lambda B^3 t^{-3\mu} = 0. \tag{87}$$

Considering terms of order $\mathcal{O}(t^{-\mu-2})$, equation (86) gives $\xi \geq \frac{1}{12}$ if we require a positive and real ν. So for this special ansatz ξ is restricted to the condition $\frac{1}{12} \leq \xi \leq \frac{1}{6}$. With the same procedure, equation (87) gives

$$\mu^2 + (1 - 3\nu)\mu + 12\xi\nu^2 + 6\xi\nu = 0. \tag{88}$$

Again, positivity and reality of solutions for μ lead to the following constraint

$$(9 - 48\xi)\nu^2 - (6 + 24\xi)\nu + 1 \geq 0 \tag{89}$$

where for $\xi = \frac{1}{12}$ and taking equality, we find $\nu = \frac{4\pm\sqrt{11}}{5}$ which plus sign obviously leads to an power-law accelerated expansion. So, we conclude that although in the presence of non-minimally coupled scalar field, accelerated expansion of universe is harder to achieve
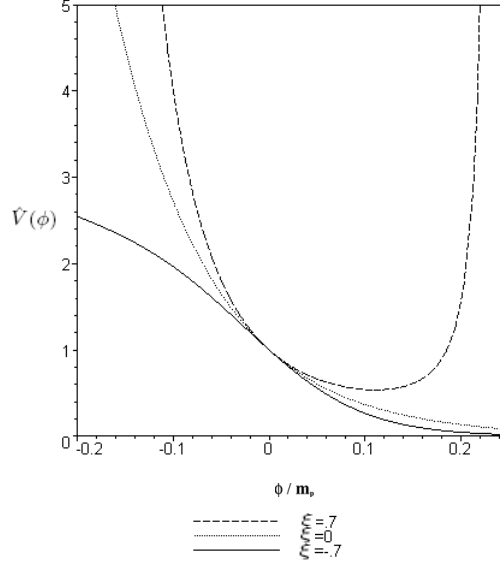
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Figure 3: An effective potential $\hat{V}(\phi)$ in the Einstein frame in the case of $\xi = 0.7, 0, -0.7$ with $p = 1/2$ (top, middle, bottom). When ξ is positive, since the potential $\hat{V}(\phi)$ is flat in the region of $\phi > 0$ compared with the $\xi = 0$ case, we can expect assisted inflation to occur in this region. When ξ is negative, assisted inflation can be realized in the region of $\phi < 0$.

relative to minimally coupled scalar field case, but with a suitable choice of nonminimal coupling it is possible to explain this accelerated expansion.

On the other hand, with a nonminimal coupling of the kind $-\frac{1}{2}\xi R\phi^2$, we can define

$$G_{eff} = \frac{G}{1 - \frac{\phi^2}{\phi_c^2}}, \tag{90}$$

where $\phi_c^2 \equiv \frac{m_3^2}{8\pi\xi}$. In order to connect to our present universe, G_{eff} needs to be positive for the case of the positive ξ, which yields $|\phi| < \phi_c = \frac{m_3}{\sqrt{8\pi\xi}}$. When ξ is negative, such a constraint is absent. By performing a conformal transformation[40] to transform to the Einstein frame, we define

$$\hat{V} \equiv \frac{V(\phi)}{(1 - \xi\kappa^2\phi^2)^2} \tag{91}$$

Figure 3 shows the behavior of this effective potential with different values of nonminimal coupling and $V(\phi)$ defined in (74). As we see, when ξ is positive, since the potential $\hat{V}(\phi)$ is flat in the region of $\phi > 0$ compared with the $\xi = 0$ case, we can expect assisted inflation (due to NMC) to occur in this region. When ξ is negative, assisted inflation can be realized in the region of $\phi < 0$.

Based on some holographic dark energy model, the value of NMC, ξ to have an accelerated universe is restricted to the interval $0.146 \preceq \xi \preceq 0.167$ [44]. On the other hand,

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current experimental limits on the time variation of G constraint the nonminimal coupling as $-10^{-2} \preceq \xi \preceq 10^{-2}$ [45]. Solar system experiments such as Shapiro time delay and deflection of light[33] have constraint Brans-Dike parameter to be $\omega_{BD} > 500$. This constraint leads to the result of $|\xi| \preceq 2.2 \times 10^{-2}$ for non-minimal coupling [45].

6   Quintessence Model

To study quintessence model within our setting, we write the equation of state for scalar field which takes the following form

$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi) + 4\left(\alpha'\ddot{\phi} + 2H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2\right)}{\dot{\phi}^2 + 2V(\phi) - 12\alpha'H\dot{\phi}}. \tag{92}$$

Causality implies that $|w| \leq 1$. When $\dot{\phi} = 0$, we obtain $p_\phi = -\rho_\phi$. In this case ρ_ϕ is independent of a and $V(\phi)$ plays the role of a cosmological constant. If $V(\phi) = 0$, we find

$$w = \frac{\dot{\phi}^2 + 4\left(\alpha'\ddot{\phi} + 2H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2\right)}{\dot{\phi}^2 - 12\alpha'H\dot{\phi}}. \tag{93}$$

In the case of minimal coupling ($\alpha = const.$) this is corresponding to a massless scalar field which plays the role of a *stiff matter* since $\rho_\phi \sim \frac{1}{a^6}$. However, in our case with non-minimally coupled scalar field, the situation is very different since now α plays a crucial role and depending on the form of α we may obtain some traces of stiff matter or fail to have such extreme case. So, in principle our model provide a mechanism to avoid stiff matter. In the minimal case when $\dot{\phi}^2 < V(\phi)$, we obtain $p_\phi < -\frac{\rho_\phi}{3}$ which shows a late-time accelerating universe. In this case, since $V(\phi) = \frac{1}{2}(1-w)\rho_\phi$ and $\phi = \sqrt{3(1+w)}\ln a$, the following potential which is a Liouville-type potential, decreases when scalar field ϕ increases and therefore gives the required quintessence

$$V(\phi) = V_0 \exp\left(-\sqrt{3(1+w)}\phi\right), \tag{94}$$

where $0 < 3(1+w) < 2$. On the other hand, for non-minimal case we find

$$V(\phi) = \frac{1}{2}\alpha(1-w)\rho + \left(\alpha'\ddot{\phi} + 5H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2\right). \tag{95}$$

In this case the quintessence potential has an explicit dependence on the non-minimal coupling and its derivatives with respect to ϕ. To have a quintessence model, we should impose some limits on the shape of this non-minimal coupling. Since based on quintessence proposal the dark energy of the universe is dominated by the potential of scalar field,

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we should impose suitable constraints on non-minimal coupling such that scalar field potential decrease when scalar field increases. We assume ordinary matter on the brane has energy density $\rho \propto a^{-3}$ and vanishing pressure $p = 0$. Now total energy density becomes $\rho_T \equiv \rho_\phi + \rho$. The necessary and sufficient condition for acceleration of the universe, $\rho_T + 3P_T < 0$, leads to the following relation

$$(1 + 3\alpha'')\dot{\phi}^2 - V(\phi) + 3\alpha'^2 R - 6\alpha' H_0 \dot{\phi} - 3\alpha' \frac{dV}{d\phi} + \frac{\rho}{2} < 0, \tag{96}$$

which can be written as

$$\rho_\phi - 2V(\phi) + (\frac{1}{2} + 3\alpha'')\dot{\phi}^2 + 3\alpha'^2 R - 3\alpha' \frac{dV}{d\phi} + \frac{\rho}{2} < 0. \tag{97}$$

This is a general constraint to have a quintessence scenario in the presence of non-minimal coupling. If we assume that ρ and ρ_ϕ are non-negative, with $\alpha(\phi) = \frac{1}{2}(1 - \xi\phi^2)$ and $\xi \leq \frac{1}{6}$, we find

$$V - \frac{3\xi}{2}\phi\frac{dV}{d\phi} > 0. \tag{98}$$

To have quintessential expansion, this constraint with $0 < \xi \leq \frac{1}{6}$, restrict the form of scalar field potential to potentials $V(\phi) > 0$ where $\frac{d}{d\phi}\left[\ln\left[\frac{V}{V_0}\left(\frac{\phi_0}{\phi}\right)^\omega \exp\left(-\frac{\phi^2}{6}\right)\right]\right] < 0$ where $\omega = \frac{1}{3\xi}\left(1 - \frac{\Omega}{2\Omega_\phi}|_{t=t_0}\right)$, $\Omega = \rho/\rho_c$, $\Omega_\phi = \rho_\phi/\rho_c$ and $\frac{\Omega}{\Omega_\phi}$ has been approximated by its present value[22]. So, to have a quintessential expansion with non-minimal coupling with $0 < \xi \leq 1/6$ we need a potential that does not grow faster than $f(\phi) = V_0\left(\frac{\phi_0}{\phi}\right)^\omega \exp\left(\frac{\phi^2}{6}\right)$ with variation of ϕ. An inverse-power-law potential such as $V(\phi) = \mu^{4+\delta}\phi^{-\delta}$, an exponential potential such as $V(\phi) = \mu^4 \exp\left(-\lambda\frac{\phi}{m_4}\right)$ and several other possibilities[38,39] provide suitable potentials for quintessence. But obviously quintessence model in the presence of non-minimal coupling needs more artificial arguments than minimal case.

7 Summary and Conclusions

In this paper we have considered the DGP model with a non-minimally coupled scalar field on the brane. As we have explained, the introduction of non-minimal coupling is not just a matter of taste; it is forced upon us in many situations of physical and cosmological interests such as quantum corrections to the scalar field theory and its renormalizability in curved spacetime. In the spirit of DGP inspired gravity, we have studied the effect of an induced gravity term which is an arbitrary function of a scalar field on the brane. We have presented four-dimensional equations on a DGP brane with a scalar field non-minimally coupled to the induced Ricci curvature, embedded in a five-dimensional Minkowski bulk.

This is an extension to a braneworld context of scalar-tensor (Brans-Dicke) gravity. Cosmological implications of both minimal and non-minimal extension of our model are studied. In minimal case, we have considered an exponentially decreasing potential which has been motivated by modified theory of gravity with R^{-1} modification. In the non-minimal case however, we have considered potential of the type $V(\phi) = \lambda \phi^n$. We have studied the weak field limit of our model and it has been shown that the mass density of ordinary matter on the brane should be modified by the addition of the effective mass density attributed to the non-minimally coupled scalar field on the brane. Also in this case crossover scale of DGP scenario is modified by the presence of non-minimal coupling. We have discussed the role of nonminimal coupling in crossover distance. Considering the case of FRW brane, we have obtained the evolution of the metric and scalar field by solving the field equations in the limit of small curvature. Our solutions for minimal case predict a power-law acceleration on the brane supporting observed late-time acceleration. For non-minimal case we have shown that by a suitable choice of non-minimal coupling and scalar field potential one can achieve accelerated expansion in some special cases. However, As Faraoni has shown, in the presence of non-minimally coupled scalar field accelerated expansion of universe is harder to achieve relative to minimally coupled scalar field case. We have studied quintessence model in our framework and it has been shown that for a restrict class of non-minimal coupling one can achieve quintessence potential. As an important achievement our analysis shows that DGP model allows for an embedding of the standard Friedmann cosmology in the sense that the cosmological evolution of the background metric on the brane can entirely be described by the standard Friedmann equation plus energy non-conservation on the brane. As we have shown, our model gives dark energy extension of Friedmann equation more easily than standard framework. Some observational and experimental constraint on nonminimal coupling are discussed. The issue of Non-minimal inflation on warped DGP braneworld and confrontation with WMAP3 data will be reported in a separated paper[46].

References

[1] A. G. Riess *et. al.* (Supernova Search Team Collaboration), Astron. J. **116** (1998) 1006, [astro-ph/9805201].

[2] S. Perlmutter *et. al.*, Astrophys. J. **517** (1999) 565, [astro-ph/9812133], D. N. Spergel *et. al.*, Astrophys. J. Suppl. **148** (2003) 175, [astro-ph/0302209].

[3] C. L. Bennett *et. al.*, Astrophys. J. Suppl. **148** (2003) 1, [astro-ph/0302207].

[4] C. B. Netterfield *et. al.*, Astrophys. J. **571** (2002) 604, [astro-ph/0104460], N. W. Halverson *et. al.*, Astrophys. J. **568** (2002) 38, [astro-ph/0104489].

[5] D. N. Spergel *et al*, *WMAP Three Year Results: Implication for Cosmology*, arXiv:astro-ph/0603499

[6] T. Padmanabhan, Phys. Rept. **380** (2003) 253
V. Shani and A. A. Astarobinsky, Int. J. Mod. Phys. D **9** (2000) 373
S. M. Carroll, Living Rev. Rel. **4** (2001) 1
E. J. Copeland, M. Sami and S. Tsujikawa, arXiv:hep-th/0603057
S. Nesseris and L. Perivolaropoulos, arXiv:astro-ph/0610092

[7] R. R. Caldwell, Phys. Lett. B **545** (2002) 23
R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys Rev. Lett. **91** (2003) 071301
J. M. Cline, S. Y. Jeon and G. D. Moore, Phys. Rev. D **70** (2004) 043543

[8] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80** (1998) 1582
P. J. E. Peebles and A. Vilenkin, Phys. Rev. D **59** (1999) 063505
P. J. Steinhardt, L. M. Wang and I Zlatev, Phys. Rev. D **59** (1999)123504
M. Doran and J. Jaechel, Phys. Rev. D **66** (2002) 043519
A. R. Liddle, P. Parson and J. D. Barrow, Phys. Rev. D **50** (1994) 7222

[9] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485** (2000) 208, [hep-th/0005016]
G. Dvali and G. Gabadadze, Phys. Rev. D **63** (2001) 065007, [hep-th/0008054],
G. Dvali, G. Gabadadze, M. Kolanović and F. Nitti, Phys. Rev. D **65** (2002) 024031, [hep-th/0106058]

[10] C. Deffayet, Phys. Lett. B **502** (2001) 199, [hep-th/0010186]

C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65** (2002) 044023, [astro-ph/0105068]

C. Deffayet and S. J. Landau, J. Raux, M. Zaldarriaga and P. Astier, Phys. Rev. D **66** (2002) 024019, [astro-ph/0201164],

J. S. Alcaniz, Phys. Rev. D **65** (2002) 123514, [astro-ph/0202492]

D. Jain, A. Dev and J. S. Alcaniz, Phys. Rev. D **66** (2002) 083511, [astro-ph/0206224]

A. Lue, R. Scoccimarro, G. Starrkman, Phys. Rev. D **69** (2004) 044005, [astro-ph/0307034].

[11] S. M. Carroll, V. Duvvuri, M. Trodden, M. Turner, Phys. Rev. D **70** (2004) 043528, [astro-ph/0306438].

[12] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **429** (1998) 263, [hep-ph/9803315]

N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Rev. D **59** (1999) 086004, [hep-th/9807344]

[13] L. Randall, R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370

L. Randall, R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690

[14] P. Binétruy, C. Deffayet, D. Langlois, Nucl. Phys. B **565** (2000) 269, [hep-th/9905012]

P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **447** (2000) 285, [hep-th/9905012]

R. Maartens, Living Rev. Relativity, **7** (2004) 7, http://www.livingreviews.org/lrr-2004-7

D. Wands, arXiv: gr-qc/0601078 and references therein.

[15] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **436** (1998) 257

D. Cremades, L. E. Ibanez and F. Marchesano, Nucl. Phys. B **643** (2002) 93, [hep-th/0205074]

C. Kokorelis, Nucl. Phys. B **677** (2004) 115, [hep-th/0207234]

[16] W. Mück, K. S. Viswanathan, I. V. Volovich, Phys. Rev. D **62** (2000) 105019, [hep-th/0004017].

[17] R. Gregory, V. A. Rubakov, S. M. Sibiryakov, Class. Quan. Grav. **17** (2000) 4437, [hep-th/0003109].

[18] I. Ya. Aref'eva, M. G. Ivanov, W. Mück, K. S. Viswanathan and I. V.Volovich, Nucl. Phys. B **590** (2000) 273, [hep-th/0004114].

[19] M. Cvetič, M. J. Duff, J.T. Liu, H. Lu, C. N. Pope and K. S. Stelle, Nucl. Phys. B **605** (2001) 141, [peh-th/0011167].

[20] R. Dick, Class. Quant. Grav. **18** (2001) R1, [hep-th/0105320]
R. Dick, Actaphys. Polon. B **32** (2001) 3669, [hep-th/0110162].

[21] H. Collins and B. Holdom, Phys. Rev. D **62** (2000) 105009, [arXiv:hep-th/0003173]
Y. V. Shtanov, arXiv:hep-th/0005193
N. J. Kim, H. W. Lee and Y. S. myung, Phys. Lett. B **504** (2001) 323, [arXiv:hep-th/0101091]

[22] V. Faraoni, Phys. Rev. D **62** (2000) 023504, [arXiv:gr-qc/0002091]

[23] S. Mizuno, Kei-ichi Maeda and K. Yamamoto, Phys. Rev. D **67** (2003) 023516, [hep-th/0205292]

[24] Kei-ichi Maeda, S. Mizuno and T. Torii, Phys. Rev. D **68** (2003) 024033, [arXiv:gr-qc/0303039]

[25] S. C. Davis, JHEP **03** (2002) 058, [arXiv:hep-ph/0111351 and arXiv:hep-th/0106271]

[26] C. Bogdanos, A. Dimitriadis and K. Tamvakis, arXiv:hep-th/0611181

[27] A. Dimitriadis, C. Bogdanos and K. Tamvakis, Phys. Rev. D **74** (2006) 045003

[28] K. Farakos and P. Pasipoularides, Phys. Lett. B **621** (2005) 244
K. Farakos and P. Pasipoularides, Phys. Rev. D **73** (2006) 084012, [arXiv:hep-th/0610010].

[29] Rong-Gen Cai and H. Zhang, JCAP **0408** (2004) 017, [arXiv:hep-th/0403234]

[30] K. Atazadeh and H. R. Sepangi, Phys. Lett. B **643** (2006) 76, [arXiv:gr-qc/0610107]

[31] M. Bouhamdi-Lopez and D. Wands, Phys. Rev. D **71** (2005) 024010,[arXiv:hep-th/0408061]

[32] J. W. York, Phys. Rev. Lett. **28** (1972) 1082
G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15** (1977) 2752

[33] S. M. Carroll, *An Introduction to General Relativity: Spacetime and Geometry*, Addison Wesley, 2004

[34] A. Lue, Physics Reports **423** (2006) 1-48, [astro-ph/0510068]

[35] V. Faraoni, *Phys. Rev. D* **53** (1996) 6813

[36] A. Bonanno, Phys. Rev. D **52** (1995) 969; A. Bonanno and D. Zappalá, *ibidem* **55** (1997) 6135

[37] C. T. Hill and D. S. Salopek, Ann. Phys. (NY) **213** (1992) 21

[38] S. Mizuno and Kei-ichi Maeda, *Phys. Rev. D* **64** (2001) 123521

[39] V. Sahni, *Chaos, Solitons and Fractals*, **16** (2003) 527

[40] S. Tsujikawa, *Phys. Rev. D* **62** (2000) 043512

[41] C. Deffayet, G. R. Dvali, G. Gabadadze, *Phys. Rev. D* **65** (2002) 044023

[42] C. Deffayet *et al*, *Phys. Rev. D* **66** (2002) 024019

[43] [Supernova Search Team Collaboration], A.G. Riess *et al.*, Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: evidence for past deceleration and constraints on dark energy evolution, *Astrophys. J.* **607** (2004) 665.

[44] M. Ito, *Europhys. Lett.* **71** (5) (2005) 712

[45] T. Chiba, *Phys. Rev. D* **60** (1999) 083508

[46] K. Nozari and B. Fazlpour, work in progress